

On Spontaneous Baryogenesis and Transport Phenomena

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Abstract

The spontaneous baryogenesis mechanism by Cohen, Kaplan and Nelson is reconsidered taking into account the transport of particles inside the electroweak bubble walls. Using linear response theory, we calculate the modifications on the thermal averages of the charges of the system due to the presence of a space time dependent ‘charge potential’ for a quantum number not orthogonal to baryon number. The local equilibrium configuration is discussed, showing that, as a consequence of non zero densities for conserved charges, the $B + L$ density is driven to non zero values by sphaleronic processes.

Solving a rate equation for the baryon number generation, we obtain an expression for the final baryon asymmetry of the Universe containing the relevant parameters of the bubble wall, *i.e.* its velocity, its width, and the width of the region in which sphalerons are active. Compared to previous estimates in which transport effects were not taken into account, we find an enhancement of nearly three orders of magnitude in the baryon asymmetry.

Finally, the role of QCD sphalerons in cooperation with transport effects is analyzed, showing that the net result depends crucially on the particle species which enter into the charge potential.

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1. Introduction

The possibility of a baryogenesis at the electroweak scale is a very popular but controversial topic. Despite the large number of related publications [1], none of the key aspects of this subject can be considered to be on firm ground. First, it is well known that the requirement that the anomalous ‘sphaleronic’ processes which violate baryon number (B) go out of equilibrium soon after the transition translates into a lower bound on the ratio between the vacuum expectation value (VEV) of the Higgs field, $v(T)$, and the critical temperature of the transition, T_c , $v(T_c)/T_c \gtrsim 1$. In the standard model, improved perturbative evaluations of this ratio [2] give a value which is badly less than unity for values of the mass of the Higgs scalar compatible with LEP results. The situation in the minimal supersymmetric standard model is slightly better, however the allowed region of the parameter space is very small, and is likely to be excluded by future LEP data [3]. On the other hand, the perturbative expansion cannot be trusted any more for values of the Higgs masses of the order of the W boson mass or larger [4], and preliminary results based on non perturbative methods (lattice simulations [5], $1/\varepsilon$ expansion [6], effective action [7]) give indications of strong differences of the results with respect to those obtained perturbatively. Clearly, much work is still needed on this issue.

Another aspect of the problem is CP violation. This has been the subject of a recent debate in the literature about the need of further complex phases in the theory besides the one in the Cabibbo-Kobayashi-Maskawa mixing matrix (see. [8]). In models with more than one Higgs doublet, like the minimal supersymmetric standard model, further sources of CP violation can emerge naturally from the Higgs sector. In particular, the possibility of a spontaneous CP violation at finite temperature has been emphasized [9, 10, 11]. This effect could give enough contribution for the baryogenesis and at the same time satisfy the upper bounds on CP violation coming from the electric dipole moment of the neutron.

Even assuming that the phase transition is strong and that CP violation is enough, we must face the other key issue, namely what is the mechanism responsible for the generation of baryons during the phase transition. The most significant departure from thermodynamic equilibrium takes place at the passage of the walls of the expanding bubbles which convert the unbroken into the broken phase. According to the size and speed of the bubble walls, two different mechanisms are thought to be dominant. In the case of “thin” (width $\sim 1/T$) walls, typical of a very strong phase transition, the creation of baryons occurs via the asymmetric (in baryon number) reflection of quarks from the bubble wall, which biases the sphaleronic transitions in the region in front of the expanding bubble [12]. If the walls are “thick” (width $\sim (10 - 100)/T$) then the relevant mechanism takes place inside the bubble walls rather than

in front of them. In this case we can make a distinction between ‘fast’ processes (mediated by gauge, flavour diagonal, interactions and by top Yukawa interactions) and ‘slow’ processes (mediated by Cabibbo suppressed gauge interactions and by light quarks Yukawa interactions). The former are able to follow adiabatically the changing of the Higgs VEV inside the bubble wall, while, in first approximation, the latter are frozen during the passage of the wall. If CP violation, explicit or spontaneous, is present in the scalar sector then a space-time dependent phase for the Higgs VEVs is turned on inside the wall. The time derivative of this phase couples with the density of a quantum number non orthogonal to baryon number (e.g. fermion hypercharge density)⁴ and then can be seen as an effective chemical potential, named “charge potential”, which has the effect of biasing the rates of the sphaleronic processes, creating an asymmetry proportional to $\dot{\vartheta}$, where ϑ is the phase of the VEVs.

This “adiabatic scenario”, originally due to Cohen, Kaplan, and Nelson (CKN) [14], has been recently reconsidered by different authors in different but related aspects. First, Giudice and Shaposhnikov have shown the dramatic effect of non perturbative, chirality breaking, transitions induced by the so called “QCD sphalerons” [15]. If these processes were active inside the bubble walls, then the equilibrium value for baryon number in the adiabatic approximation would be proportional to that for the conserved quantum number $B - L$ (L is the lepton number), up to mass effects suppressed by $\sim (m_{top}(T)/\pi T)^2$. Then, imposing the constraint $\langle B - L \rangle = 0$ (here $\langle \dots \rangle$ represents the thermal average) we obtain zero baryon number (up to mass effects).

Dine and Thomas [16] have considered the two Higgs doublets model in which the same doublet couples both to up and down quarks, the same model considered in the original work by CKN. These authors have pointed out that $\dot{\vartheta}$ couples also to the Higgs density, so that the induced charge potential is for total hypercharge rather than for fermion hypercharge. As long as effects proportional to the temperature dependent VEV $v(T)$ are neglected, hypercharge is an exactly conserved quantum number and then, again imposing the constraint that all the conserved charges have zero thermal averages, no baryon asymmetry can be generated. So, we again find a $m_{top}(T)^2/T^2$ suppression factor.

Finally, Joyce, Prokopec and Turok (JPT) have emphasized the very important point that the response of the plasma to the charge potential induced by $\dot{\vartheta}$ is not simply that of a system of fixed charges, because transport phenomena may play a crucial role [17]. When a space time dependent charge potential is turned on at a certain point, hypercharged particles are displaced from the surrounding regions, so that even the thermal averages of conserved quantum numbers become locally non vanishing. As a consequence, the equilibrium properties of the system have to be reconsidered taking into account the local ‘violation’ of the conserved quantum numbers.

⁴The phase also couples to Chern-Simons number but this coupling induces an effect which is suppressed by $m_{top}(T)^2/T^2$ with respect to the one which we are presently discussing [13], so we will neglect it.

In this paper we analyze the adiabatic scenario using linear response theory [18, 23] in order to take transport effects into account. We assume that a spacetime dependent charge potential for fermion hypercharge is generated inside the bubble wall, without discussing its origin, and investigate its effect on the thermal averages of the various quantum numbers of the system. We find that transport phenomena are really crucial, but we disagree with JPT's conclusion that as a consequence of the local 'violation' of global quantum numbers there is no biasing of the sphaleronic processes. Actually, in the adiabatic approximation the local equilibrium configuration of the system is determined by the thermal averages of the charges conserved by all the fast interactions. The effect of transport phenomena is to induce space time dependent non zero values for these averages. We calculate these averages using linear response theory and then determine the local equilibrium configuration, showing that it corresponds to $\langle B + L \rangle \neq 0$. As we will discuss, JPT's result corresponds to freezing out any interaction inside the bubble wall, which is in contradiction with the adiabatic hypothesis. Then we write down a rate equation in order to take into account the slowness of the sphaleron transitions and obtain an expression for the final baryon asymmetry explicitly containing the parameters describing the bubble wall, such as its velocity, v_w , its width, and the width of the region in which the sphalerons are active.

The inclusion of transport phenomena also sheds a new light on the strong sphaleron effects and on the effect of a charge potential for total rather than fermionic hypercharge. The dramatic suppressions found by Giudice and Shaposhnikov and by Dine and Thomas respectively, are both a consequence of taking zero averages for conserved quantum numbers. Since these averages are no more locally zero we will find a non zero $\langle B + L \rangle \neq 0$, even in the case in which the charge potential is for total rather than for fermion hypercharge. In the case of QCD sphalerons we find that the final result depends in a crucial way on the form of the charge potential which is considered. For example, if all the left handed fermions plus the right handed quarks contributed to the charge potential according to their hypercharge, then no bias of sphaleron processes would be obtained. In this case, we would find a non zero value for $B+L$ inside the bubble wall but no baryon asymmetry far from it inside the broken phase. On the other hand, if only right and left handed top quarks participate to the charge potential, then a final asymmetry is found and QCD sphalerons are harmless.

The plan of the paper is as follows: in section 2 we will develop a chemical potential analysis for the equilibrium properties of the plasma inside the bubble wall in the adiabatic approximation, and we will write down the rate equation for the production of $B + L$ due to sphaleron transitions; in sect. 3 we will introduce our application of linear response analysis to the calculation of the variations of the thermal averages induced by $\dot{\vartheta}$. In particular we will show that the fundamental quantity to evaluate is the retarded two points Green's function for

fermion currents. In section 4 we will solve the rate equation, finding an expression for the final baryon asymmetry in terms of the various bubble wall parameters. In this context, we will also discuss the screening effects on the electric charge. In sect. 5 we will discuss the role of QCD sphalerons in cooperation with transport phenomena, and finally we will discuss our results in section 6.

2. Local equilibrium inside the wall

For definiteness, we will work in the two Higgs doublet model in which one doublet couples to up quarks and the other one to down quarks. The phase transition is assumed to be strong enough so that sphaleron processes freeze out somewhere inside the bubble wall (see the end of sect. 4 and ref. [16] for a discussion about this point).

The relevant timescale for baryogenesis is given by the passage of the bubble wall, which takes $\Delta t_w = \Delta z/v_w \simeq (200)/T$ [19], where Δz is the wall thickness. During this time the phase of the Higgs VEV's changes of an amount $\Delta\vartheta$. Thus, we can discriminate between fast processes, which have a rate $\gtrsim 1/\Delta t_w$ and then can equilibrate adiabatically with $\dot{\vartheta}$, and slow interactions, which feel that ϑ is changing only when the bubble has already passed by and sphalerons are no more active. Next, we introduce a chemical potential for any particle which takes part to fast processes, and then reduce the number of linearly independent chemical potentials by solving the corresponding system of equations, in a way completely analogous to that followed for example in refs. [20], the main difference here being that light quark Yukawa interactions and Cabibbo suppressed gauge interactions are out of equilibrium. Finally, we can express the abundances of any particle in equilibrium in terms of the remaining linear independent chemical potentials, corresponding to the conserved charges of the system.

Since strong interactions are in equilibrium inside the bubble wall, and since the current coupled to $\dot{\vartheta}$ is color singlet, we can chose the same chemical potential for quarks of the same flavour but different color, and set to zero the chemical potential for gluons. Moreover, since inside the bubble wall $SU(2) \times U(1)$ is broken, the chemical potential for the neutral Higgs scalars vanishes⁵.

The other fast processes, and the corresponding chemical potential equations are:

i) top Yukawa:

$$\begin{aligned} t_L + H_2^0 &\leftrightarrow t_R + g & (\mu_{t_L} = \mu_{t_r}) \\ b_L + H^+ &\leftrightarrow t_R + g & (\mu_{t_R} = \mu_{b_L} + \mu_{H^+}) \end{aligned} \tag{1}$$

⁵This is true if chirality flip interactions, or processes like $Z \rightarrow Z^*h$, are sufficiently fast; since the corresponding rates depend on the Higgs VEV, they will be suppressed by factors of $(v(T)/T)^2$ with respect for example to the rate for $ht_L \leftrightarrow t_Rg$. This has led the authors of refs.[15, 17] to consider the system in the unbroken phase. Anyway, this choice does not lead to dramatic changes to the conclusion of this paper.

ii) SU(2) flavour diagonal:

$$\begin{aligned}
e_L^i &\leftrightarrow \nu_L^i + W^- & (\mu_{\nu_L^i} &= \mu_{e_L^i} + \mu_{W^+}) \\
u_L^i &\leftrightarrow d_L^i + W^+ & (\mu_{u_L^i} &= \mu_{d_L^i} + \mu_{W^+}) \\
H_2^0 &\leftrightarrow H^+ + W^- & (\mu_{H^+} &= \mu_{W^+}) \\
H_1^0 &\leftrightarrow H^- + W^+ & (\mu_{H^-} &= -\mu_{W^+})
\end{aligned} \quad (i = 1, 2, 3) \quad (2)$$

Neutral current gauge interactions are also in equilibrium, so we have zero chemical potential for the photon and the Z boson.

Imposing the above constraints, we can reduce the number of independent chemical potentials to four, μ_{W^+} , μ_{t_L} , $\mu_{u_L} \equiv 1/2 \sum_{i=1}^2 \mu_{u_L^i}$, and $\mu_{e_L} \equiv 1/3 \sum_{i=1}^3 \mu_{e_L^i}$. These quantities correspond to the four linearly independent conserved charges of the system. Choosing the basis Q' , $(B-L)'$, $(B+L)'$, and $BP' \equiv B'_3 - 1/2(B'_1 + B'_2)$, where the primes indicate that only particles in equilibrium contribute to the various charges, and introducing the respective chemical potentials, we can go to the new basis using the relations

$$\begin{cases} \mu_{Q'} &= 3\mu_{t_L} + 2\mu_{u_L} - 3\mu_{e_L} + 11\mu_{W^+} \\ \mu_{(B-L)'} &= 3\mu_{t_L} + 4\mu_{u_L} - 6\mu_{e_L} - 6\mu_{W^+} \\ \mu_{(B+L)'} &= 3\mu_{t_L} + 4\mu_{u_L} + 6\mu_{e_L} \\ \mu_{BP'} &= 3\mu_{t_L} - 2\mu_{u_L} \end{cases} \quad (3)$$

If sphaleron transitions were fast, then we could eliminate a further chemical potential through the constraint

$$2 \sum_{i=1}^3 \mu_{u_L^i} + \sum_{i=1}^3 \mu_{d_L^i} + \sum_{i=1}^3 \mu_{e_L^i} = 0. \quad (4)$$

In this case, the value of $(B+L)'$ would be determined by that of the other three charges according to the relation

$$(B+L)'_{EQ} = \frac{3}{80}Q' + \frac{7}{20}BP' - \frac{19}{40}(B-L)' \quad (5)$$

where we have indicated charge densities by the corresponding charge symbols. We have neglected mass effects, which means that the excess of particle over antiparticle density is related to the chemical potentials according to the relation [20] $n_+ - n_- = agT^3/6(\mu/T)$, where $a = 1$ for fermions and $a = 2$ for bosons, and g is the number of spin and color degrees of freedom.

The above result should not come as a surprise, since we already know from ref. [20] that a non zero value for $B-L$ gives rise to a non zero $B+L$ at equilibrium. Stated in other words, sphaleron transitions erase the baryon asymmetry only if any conserved charge of the system has vanishing thermal average, otherwise the equilibrium point lies at $(B+L)_{EQ} \neq 0$.

Actually, sphaleron rates are too small to allow $(B+L)'$ to reach its equilibrium value (5), $\tau_{sp} \simeq (\alpha_W^4 T)^{-1} \gg \Delta t_W$, so equilibrium thermodynamics cannot be used to describe baryon

number generation inside the bubble wall. Following refs. [21, 14] we shall make use of the rate equation

$$\frac{d}{dt}(B+L)'_{SP} = -\frac{\Gamma_{SP}}{T} \frac{\partial F}{\partial (B+L)'} \quad (6)$$

where $\Gamma_{SP} = k(\alpha_W T)^4 \exp(-\phi/g_W T)$ is the rate of the sphaleron transitions when the value of the Higgs field is ϕ ($k \simeq (0.1 - 1)$ from numerical simulations [22]), F is the free energy of the system, and the derivative with respect to $(B+L)'$ must be taken keeping Q' , $(B-L)'$, and BP' , constant. The meaning of eq. (6) is straightforward. Sphaleron transitions (which change $(B+L)'$ but conserve Q' , $(B-L)'$, and BP') will be turned on only if they allow the *total* free energy of the system to get closer to its minimum, *i.e.* equilibrium, value. At high temperature ($\mu_i \ll T$) the free energy of the system is given by

$$F = \frac{T^2}{12} \left[3\mu_{e_L}^2 + 3\mu_{\nu_L}^2 + 6\mu_{u_L}^2 + 3\mu_{t_L}^2 + 3\mu_{t_R}^2 + 6\mu_{d_L}^2 + 3\mu_{b_L}^2 + 6\mu_{W^+}^2 + 2\mu_{H^+}^2 + 2\mu_{H_1^0}^2 + 2\mu_{H_2^0}^2 \right]. \quad (7)$$

Using (1), (2) and (3) to express the chemical potentials in terms of the four conserved charges in (3) we obtain the free energy as a function of the density of $(B+L)' = \mu_{B+L} T^2/6$,

$$F[(B+L)'] = 0.46 \frac{[(B+L)' - (B+L)'_{EQ}]^2}{T^2} + \text{constant terms} \quad (8)$$

where the “*constant terms*” depend on Q' , $(B-L)'$, and BP' but not on $(B+L)'$, and $(B+L)'_{EQ}$ is given by (5). The total amount of $(B+L)'$ present in a certain point at a certain time is made up by two contributions: $(B+L)'_{SP}$, generated by sphaleron transitions, and $(B+L)'_{TR}$, which is not generated but is transported from nearby regions in response to the perturbation introduced by $\dot{\vartheta}$. So, eq. (6) now takes the form

$$\frac{d}{dt}(B+L)'_{SP} = -0.92 \frac{\Gamma_{SP}}{T^3} [(B+L)'_{SP} + (B+L)'_{TR} - (B+L)'_{EQ}] \quad (9)$$

with the initial condition $(B+L)'_{SP} = 0$ before $\dot{\vartheta}$ is turned on.

Let us summarize our discussion up to this point. Consider an observer in the plasma reference frame during the phase transition. When a bubble wall passes by the observer, he measures a space time dependent charge potential for, say, fermionic hypercharge, which induces transport phenomena and then local asymmetries in particle numbers. The Q' , $(B-L)'$, and BP' components of these asymmetries remain unaffected by fast interactions, while the other components are reprocessed as to obtain their equilibrium values. In the case of $(B+L)'$ the reprocessing is slow, so we have to use the rate equation in (9) to describe it. The generation of $(B+L)'_{SP}$ will go on until either Γ_{SP} goes to zero or the local equilibrium value is reached *i.e.* $(B+L)'_{SP} + (B+L)'_{TR} = (B+L)'_{EQ}$. After the passage of the wall, $(B+L)'_{EQ}$ and $(B+L)'_{TR}$

go rapidly to zero since $\dot{\vartheta}$ vanishes, and so the final asymmetry is given by the $(B + L)'_{SP}$ generated until that time.

As we can see, the crucial question is now to calculate the induced values for Q' , $(B - L)'$, BP' , and $(B + L)'_{TR}$. We will do that in the next section by using linear response analysis [18, 23].

3. Linear response analysis

In this paragraph we want to discuss the effect on the thermal averages of the term induced in the lagrangian when $\dot{\vartheta}$ is active, which we assume to have the form

$$\mathcal{L} \rightarrow \mathcal{L} + \dot{\vartheta} J_{Y_F}^0, \quad (10)$$

with

$$J_{Y_F}^0 = \sum_i' y_F^i J_i^0 \quad (11)$$

where \sum_i' means that the sum extends on particles in equilibrium with $\dot{\vartheta}$ only. The standard procedure [14] is to consider $\dot{\vartheta}$ as an effective chemical potential, so that particle abundances at equilibrium are given by

$$\rho_i = q_i \mu_Q + (b - l)_i \mu_{B-L} + b p_i \mu_{BP} + y_i^F \dot{\vartheta}, \quad (12)$$

and then imposing

$$\langle Q' \rangle = \langle (B - L)' \rangle = \langle BP' \rangle = 0 \quad (13)$$

so that any particle abundance can be expressed in terms of $\dot{\vartheta}$ only. The point is that taking transport phenomena into account, the above thermal averages are not zero, but depend on $\dot{\vartheta}$ themselves. So we must first *calculate* their values and then use them to determine the chemical potentials. Our starting point is the generating functional

$$Z[J_O; \dot{\vartheta}] = \int_{P(A)BC} \mathcal{D}\phi \exp \left\{ i \int_C d\tau \int_V d^3\vec{x} [\mathcal{L} + \dot{\vartheta} J_{Y_F}^0 + J_O O + \text{'sources'}] \right\} \quad (14)$$

where $\mathcal{D}\phi$ is the integration measure, O is the operator of which we want to calculate the thermal average and J_O the corresponding source. C is any path in the complex τ plane connecting the point τ_{in} to $\tau_{out} = \tau_{in} - i\beta$ ($\beta = 1/T$) such that the imaginary part of τ is never increasing on the path [24]. P(A)BC means that periodic (antiperiodic) boundary conditions must be imposed on bosonic (fermionic) fields on the path.

The thermal average of the operator $O(\tau, \vec{x})$ in presence of $\dot{\vartheta}$ is obtained in the usual way

$$\langle O(t_x, \vec{x}) \rangle_{\dot{\vartheta} \neq 0} = \frac{1}{i} \frac{\delta Z[J_O; \dot{\vartheta}]}{\delta J_O(t_x, \vec{x})} \Big|_{J_O=0}. \quad (15)$$

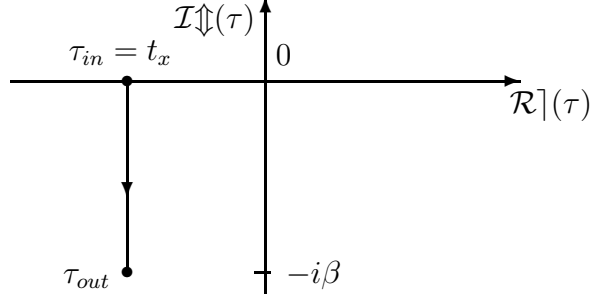


Figure 1: The path C corresponding to imaginary time formalism of thermal field theory.

where all the field sources are set to zero. Now we make a functional expansion of (15) in $\dot{\vartheta}$ and truncate it at the linear term,

$$\begin{aligned}
\langle O(t_x, \vec{x}) \rangle_{\dot{\vartheta} \neq 0} &= \frac{1}{i} \frac{\delta}{\delta J_O(t_x, \vec{x})} \left\{ Z[J_O; \dot{\vartheta} = 0] \right. \\
&\quad \left. + \int_C d\tau' \int_V d^3\vec{y} \dot{\vartheta}(\tau', \vec{y}) \frac{\delta Z[J_O; \dot{\vartheta}]}{\delta \dot{\vartheta}(\tau', \vec{y})} \Big|_{\dot{\vartheta}=0} + \dots \right\} \Big|_{J_O=0} \\
&= \langle O(t_x, \vec{x}) \rangle_{\dot{\vartheta}=0} + i \int_C d\tau' \int_V d^3\vec{y} \dot{\vartheta}(\tau', \vec{y}) \langle T_C J_{Y_F}^0(\tau', \vec{y}) O(t_x, \vec{x}) \rangle_{\dot{\vartheta}=J_O=0}
\end{aligned} \tag{16}$$

where T_C is the ordering along the path C .

Different choices for the ‘time’ contour C lead to different formulations of thermal field theory. One possibility is to take the vertical line connecting t_x to $t_x - i\beta$, so that $\tau' - t_x$ is pure imaginary on any point of the path.

This choice corresponds to the imaginary time formalism (ITF) of thermal field theory [24], and in this case we have to evaluate the euclidean two point thermal Green’s function $\langle T J_{Y_F}^0(z_E), O(0) \rangle$ ($z_E^2 = -z_0^2 - |\vec{z}|^2$). This can be done in Matsubara formalism, where Feynman rules are straightforwardly obtained [23]. But, in this case, we would have to calculate $\dot{\vartheta}$ for complex times, whereas we are interested in its values at real times, during the passage of the wall. So, in order to get a more direct physical interpretation of what we are calculating, we must turn to real time formalism. This corresponds to choose the path in Fig. 2, and then letting τ_{in} going to $-\infty$, and t_F to $+\infty$ [24]. Now, it is possible to see that the contributions to the integral coming from τ' on C_3 vanishes in the above limit [24], so we are left with the contributions from C_1 and C_2 only. The T_C ordering now allows us to rewrite the integral in

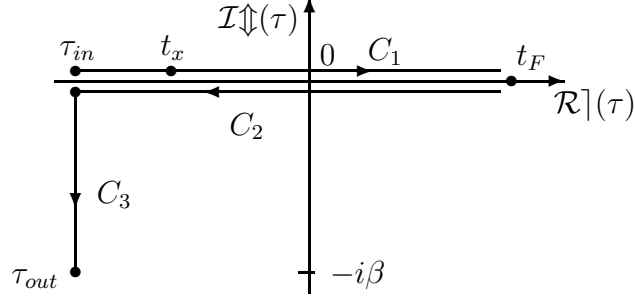


Figure 2: The path C corresponding to real time formulation of thermal field theory. C_2 lies infinitesimally beneath the real axis.

(16) as

$$\begin{aligned}
& i \int_{C_1 \oplus C_2} dt_y \int_V d^3 \vec{y} \dot{\vartheta}(t_y, \vec{y}) \langle T_C J_{Y_F}^0(t_y, \vec{y}) O(t_x, \vec{x}) \rangle_{\dot{\vartheta}=J_O=0} \\
& = i \int_{-\infty}^{t_x} dt_y \int_V d^3 \vec{y} \dot{\vartheta}(t_y, \vec{y}) \langle [J_{Y_F}^0(t_y, \vec{y}), O(t_x, \vec{x})] \rangle_{\dot{\vartheta}=J_O=0}.
\end{aligned} \tag{17}$$

Defining as usual the *retarded* Green's function

$$iD_{O,Y_F}^R(t_x, \vec{x}; t_y, \vec{y}) \equiv \langle [O(t_x, \vec{x}), J_{Y_F}^0(t_y, \vec{y})] \rangle \Theta(t_x - t_y) \tag{18}$$

where $\Theta(x)$ is the step function, we arrive at the result

$$\langle O(t_x, \vec{x}) \rangle_{\dot{\vartheta} \neq 0} = \langle O(t_x, \vec{x}) \rangle_{\dot{\vartheta}=0} + \int_{-\infty}^{+\infty} dt_y \int_V d^3 \vec{y} \dot{\vartheta}(t_y, \vec{y}) D_{O,Y_F}^R(t_x, \vec{x}; t_y, \vec{y}). \tag{19}$$

The operators we are interested in are fermionic charge densities $(Q', (B-L)', (B+L)', BP')$ of the form $Q'_A = \sum_i q_i^A J_i^0$. Inserting it in (19), and using definition (11) we get

$$\langle Q'_A(t_x, \vec{x}) \rangle_{\dot{\vartheta} \neq 0} = \sum_{ij} q_i^A y_i^F \int_{-\infty}^{+\infty} dt_y \int_V d^3 \vec{y} \dot{\vartheta}(t_y, \vec{y}) D_{ij}^R(t_x, \vec{x}; t_y, \vec{y}) \tag{20}$$

where D_{ij}^R is the current-current retarded Green's functions for fermion i and j (i and j are flavour and color indices) and we used the fact that $\langle Q'_A(t_x, \vec{x}) \rangle_{\dot{\vartheta}=0} = 0$. Note that the Green's function has to be evaluated for $\dot{\vartheta} = 0$, *i.e.* we must use the unperturbed lagrangian with the charge potential turned off and all the chemical potentials equal to zero in the partition function.

The problem of calculating the effect of the charge potential in (10) on the thermal averages for the particles in equilibrium is then reduced to the evaluation of the retarded Green's functions which enter in (20). As we have discussed, the more natural framework for this calculation is real time thermal field theory, in which the physical sense of the various quantities is evident. Anyway, we have also seen that we can calculate the euclidean Green's function in the imaginary time formalism and then continue analytically to real times, thus obtaining the D_{ij}^R 's (this

relation was established for the first time by Baym and Mermyn [25]). In energy-momentum space the analytical continuation is accomplished by the substitution

$$i\omega_n \rightarrow \omega + i\varepsilon \quad \varepsilon \rightarrow 0^+ \quad (21)$$

where $\omega_n = 2\pi nT$ are Matsubara frequencies and ω is the real energy.

4. Solution of the rate equation

Since the rate of the sphaleronic transition is suppressed by α_w^4 , the asymmetry in $(B+L)'$ generated by the sphalerons, $(B+L)'_{SP}$, is generally much smaller than both $(B+L)'_{EQ}$ and $(B+L)'_{TR}$ (we can check it *a posteriori*), so we can approximate the rate equation in (9) by

$$\frac{d}{dt}(B+L)'_{SP} \simeq 0.92 \frac{\Gamma_{SP}}{T^3} [(B+L)'_{EQ} - (B+L)'_{TR}]. \quad (22)$$

Using the equations (5) and (20) we can determine $(B+L)'_{LR} \equiv (B+L)'_{EQ} - (B+L)'_{TR}$ as

$$(B+L)'_{LR}(t_x, \vec{x}) = \langle J_{(B+L)'}^0(t_x, \vec{x}) \rangle = \sum'_{ij} c_{ij} \int_{-\infty}^{+\infty} dt_y \int_V d^3\vec{y} \dot{\vartheta}(t_y, \vec{y}) D_{ij}^R(t_x, \vec{x}; t_y, \vec{y}) \quad (23)$$

where

$$c_{ij} \equiv y_j^F \left(\frac{3}{80} q_i + \frac{7}{20} b p_i - \frac{19}{40} (b-l)_i - (b+l)_i \right).$$

Integrating eq. (22) in time from $-\infty$ to $+\infty$ we get the final density of $(B+L)'$,

$$\begin{aligned} \Delta(B+L)'_{SP} &= \int_{-\infty}^{+\infty} dt_x \frac{d}{dt_x} (B+L)'_{SP}(t_x, \vec{x}) \\ &= 0.92 \frac{2\pi}{T^3} \int_{-\infty}^{+\infty} d\omega \tilde{\Gamma}_{SP}(-\omega, \vec{x}) (\widetilde{B+L})'_{LR}(\omega, \vec{x}) \end{aligned} \quad (24)$$

where $\tilde{\Gamma}_{SP}(-\omega, \vec{x})$ and $(\widetilde{B+L})'_{LR}(\omega, \vec{x})$ are the Fourier transformed of $\Gamma_{SP}(t_x, \vec{x})$ and $(B+L)_{LR}(t_x, \vec{x})$ with respect to time.

We recall that Γ_{SP} is $k(\alpha_W T)^4$ in the unbroken phase and decreases exponentially fast as the Higgs VEV is turned on. In order to solve eq. (22) analytically we approximate this behaviour by a step function. Moreover, we will consider a plane bubble wall moving along the z -axis with velocity v_w . So, our expression for Γ_{SP} will be

$$\Gamma_{SP}(t_x, \vec{x}) \simeq \Gamma \Theta \left(t_x - t_1 - \frac{z_x}{v_w} \right) \Theta \left(t_2 - t_x + \frac{z_x}{v_w} \right) \quad (25)$$

with $t_1 \rightarrow -\infty$ and $\Gamma = k(\alpha_W T)^4$. Of course, more sophisticated approximations for Γ_{SP} may be used, at the price of solving eq. (22) numerically.

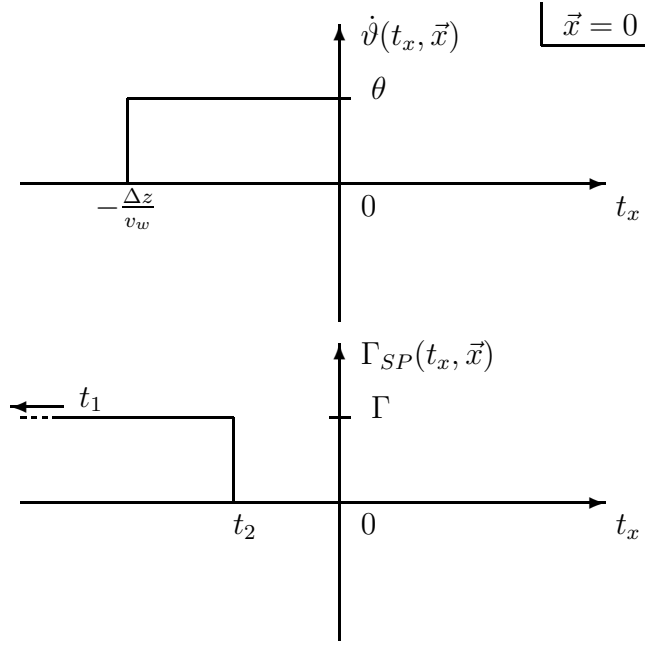


Figure 3: Schematic representation of the behaviour of $\dot{\vartheta}$ and of the rate of the sphaleronic transitions at the point $\vec{x} = 0$.

Analogously, we approximate $\dot{\vartheta}$ in such a way that it is constant in a region of width Δz inside the bubble wall, and is zero outside,

$$\dot{\vartheta}(t_y, \vec{y}) = \theta \Theta(z_y - v_w t_y) \Theta(v_w t_y - z_y + \Delta z) \quad (26)$$

where $\theta = v_w \Delta\vartheta/\Delta z$. So, if we are at the point $\vec{x} = 0$, we observe an interaction of the form (10) turned on from $t = -\Delta z/v_w$ to $t = 0$, while the sphalerons are active till $t = t_2$, as we have shown in Fig. 3.

Putting all together we obtain

$$\Delta(B + L)'_{SP} = 0.92 \frac{(2\pi)^3}{T^3} \Gamma \theta \Sigma'_{ij} c_{ij} \int_{-\infty}^{+\infty} d\omega \frac{e^{-i\omega t_2} - e^{-i\omega t_1}}{\omega} \frac{1 - e^{-i\omega \Delta z/v_w}}{\omega} \widetilde{D}_{ij}^R(p_x = p_y = 0, p_z = \frac{\omega}{v_w}, \omega). \quad (27)$$

Note the peculiar relationship between p_z and ω in the argument of \widetilde{D}_{ij}^R , due to the spacetime dependence of $\dot{\vartheta}(t_y, \vec{y})$, see (26). A consideration of the general properties of retarded Green's functions [26] ensures that the imaginary part of \widetilde{D}_{ij}^R is a even function of ω , while its imaginary part is odd. As a consequence the integral in (27) will always give a real result.

The lowest order contribution to \widetilde{D}_{ij}^R comes from the fermion loop in Fig. 4, where the two crosses indicate the zero components of the fermion current. We may evaluate the corresponding

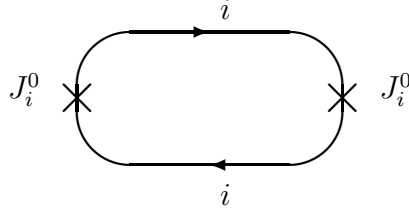


Figure 4: Lowest order contribution to D_{ij}^R

euclidean two point Green's function in the ITF and then continue analytically to real energies according to eq. (21). Moreover, since the relevant frequencies at which \widetilde{D}_{ij}^R must be evaluated are such that $\omega \leq |\vec{p}| \leq v_w/\Delta z \simeq T/10^2$ we take only the leading terms in the high temperature expansion. These are given by [27]

$$\Pi_l(p_0 = 2n\pi T, \vec{p}) \delta_{ij} = \frac{T^2}{3} \left[1 - \frac{ip_0}{2|\vec{p}|} \log \frac{ip_0 + |\vec{p}|}{ip_0 - |\vec{p}|} \right] \delta_{ij} \quad (28)$$

where we have neglected fermion masses. After continuing analytically to real energies and fixing the momenta as in (27) we get the lowest order contribution to \widetilde{D}_{ij}^R ,

$$\widetilde{D}_{ij}^{R0}(p_x = p_y = 0, p_z = \frac{\omega}{v_w}, \omega) = \frac{T^2}{3} \left[1 - \frac{v_w}{2} \log \frac{1 + v_w}{1 - v_w} + i\frac{\pi}{2} v_w \text{sign}(\omega) \right] \delta_{ij}. \quad (29)$$

When $v_w \rightarrow 1$ the above expression exhibits a collinear divergence, due to the fact that the fermions in the loops are massless in our approximation. This divergence disappears when plasma masses for fermions are taken into account. However, for our purposes, since $v_w \simeq 0.2$ [19], the effects of plasma masses for fermions can be neglected [28].

Due to the constraint $p_z = \omega/v_w$ the real part of the correlation function in (29) does not depend on ω , while the imaginary part depends on its sign only. This implies that the response induced on the plasma through (29) has neither spatial nor temporal dispersion, *i.e.* inserting (29) in (20) gives rise to an induced thermal average for the charge Q_A which in any space-time point is proportional to the value of $\dot{\vartheta}$ in that point

$$\langle Q_A(t_x, \vec{x}) \rangle^0 \propto \dot{\vartheta}(t_x, \vec{x}), \quad (30)$$

In particular also $(B+L)_{TR}$ and $(B+L)_{EQ}$ receive a contribution of this form and disappear as soon as $\dot{\vartheta}$ is turned off. Inserting it into the rate equation we obtain from (27) the contribution to the asymmetry ($t_1 \rightarrow -\infty$)

$$\Delta(B+L)'_{SP}{}^0 = 0.92 \frac{(2\pi)^3}{T^3} \Gamma \theta \sum'_{ij} c_{ij} I^0(t_2) \delta_{ij} \quad (31)$$

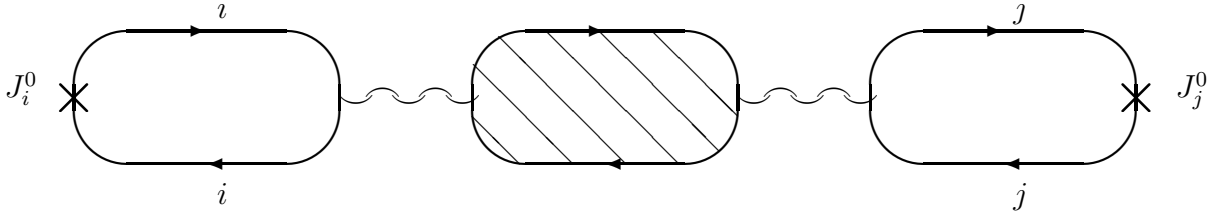


Figure 5: The contribution to D_{ij}^R due to photon exchange.

where

$$I^0(t_2) = \frac{2\pi}{3} T^2 \left(1 - \frac{v_w}{2} \log \frac{1+v_w}{1-v_w} \right) \times \begin{cases} 0 & (t_2 < -\frac{\Delta z}{v_w}) \\ \left(t_2 + \frac{\Delta z}{v_w} \right) & \left(-\frac{\Delta z}{v_w} < t_2 < 0 \right) \\ \frac{\Delta z}{v_w} & (t_2 > 0) \end{cases} \quad (32)$$

We recall that t_2 is the time at which sphaleron transitions are turned off, while the charge potential induced by $\dot{\vartheta}$ is active for $-\Delta z/v_w < t < 0$. So the asymmetry calculated in this approximation for D_{ij}^R grows linearly with t_2 until $t_2 = 0$ (for $t_2 < -\Delta z/v_w$ the asymmetry is obviously zero since there is no overlap between sphalerons and $\dot{\vartheta}$). The result for $t_2 > 0$ is an artifact of our approximation $(B+L)'_{SP} \ll (B+L)'_{LR}$, which is no more appropriate in this case. Actually, from (30) we know that when $\dot{\vartheta}$ goes to zero, as is the case for $t > 0$, $(B+L)'_{EQ}$ and $(B+L)'_{TR}$ vanish too, and the rate equation (9) becomes

$$\frac{d}{dt} (B+L)'_{SP} = -0.92 \frac{\Gamma_{SP}}{T^3} [(B+L)_{SP}] \quad (t > 0) \quad (33)$$

so that the asymmetry produced before decreases exponentially from $t = 0$ to $t = t_2$ with rate Γ . However, due to the smallness of Γ , and to the fact that t_2 cannot be much larger than $O(\Delta z/v_w)$, we can safely neglect this decreasing and take the result in (32).

Next, we consider the contribution to D_{ij}^R due to gauge bosons exchange. Since we are in the broken phase, and since the perturbation (10) induced by $\dot{\vartheta}$ is colorless, we will take into account only photons, which contribute through the graph in Fig. 5. The blob in the middle represents the sum of all possible insertion of fermion loops. In calculating the blob, we have to include not only the fermions which enter in Q' , $(B-L)'$, and BP' , *i.e.* the ones with fast flavour, or chirality, changing interactions, but we must instead take into account the contributions of all the charged fermions of the theory. This is because QED interactions are fast and then, for instance, pair production is in equilibrium also for right handed light quarks. The ‘QED’ contribution of Fig. 5 gives

$$\widetilde{D}_{ij}^{R, QED}(\omega, \vec{p}) = e^2 q_i q_j \Pi_l^2(\omega, \vec{p}) \frac{\mathcal{D}^{00}(\omega, \vec{p})}{1 - \sum_k (eq_k)^2 \mathcal{D}^{00} \Pi_l} \quad (34)$$

where eq_i is the electric charge of the fermion i . $\mathcal{D}^{00}(\omega, \vec{p})$ is the tree level $(0, 0)$ component of the photon propagator in the Coulomb gauge

$$\mathcal{D}^{\mu\nu} = \frac{-1}{p^2} P_T^{\mu\nu} - \frac{1}{|\vec{p}|^2} u^\mu u^\nu, \quad (35)$$

where $p^2 = \omega^2 - |\vec{p}|^2$, $u_\mu = (1, 0, 0, 0)$ identifies the plasma reference frame, while

$$P_T^{00} = P_T^{0i} = P_T^{i0} = 0$$

$$P_T^{ij} = \delta_{ij} - p^i p^j / |\vec{p}|^2$$

As we have discussed, the sum in the denominator of (34) must be extended over all the charged quarks and leptons. Setting $p_z = \omega/v_w$ we get

$$\widetilde{D}_{ij}^{R, QED}(\omega, \vec{p}) = -e^2 q_i q_j v_w^2 \frac{\Pi_l^2(\omega, p_z = \omega/v_w)}{\omega^2 + v_w^2 \sum_k (eq_k)^2 \Pi_l(\omega, p_z = \omega/v_w)}. \quad (36)$$

Note that unlike the ‘direct’ contribution (29) the ‘QED’ one is not flavour (or color) diagonal, so that even particles which do not enter into the expression for the charge potential (10) get a non zero thermal average depending on ϑ . Moreover, this contribution has a genuine ω dependence. Two points retarded Green’s function are analytic in the upper half of the complex ω plane. $\widetilde{D}_{ij}^{R, QED}(\omega, p_z = \omega/v_w)$ may then have poles of the form $\bar{\omega} = \omega_p - i\gamma_p$, with $\gamma_p > 0$. In order to determine them we have to solve the following equations

$$\begin{aligned} \omega_p^2 - \gamma_p^2 &= -C\mathcal{R}[\Pi_l(\bar{\omega})] \\ \omega_p &= \frac{C\mathcal{I}\Pi_l(\bar{\omega})}{2\gamma_p} \end{aligned} \quad (37)$$

where $C = v_w^2 \sum_k (eq_k)^2$. Since the RHS of the first of eqs. (37) is negative, and $v_w \simeq 0.2 < 1$, we can approximate the solutions by

$$\begin{aligned} \bar{\omega}_{1,2} &= \pm\omega_p + i\gamma_p \\ \omega_p &= \frac{\pi}{4} v_w \gamma_p \end{aligned} \quad (38)$$

$$\gamma_p \simeq [C\mathcal{R}[\Pi_l(\bar{\omega})]]^{1/2} \simeq v_w \frac{eT}{\sqrt{3}} \left(\sum_k (q_k)^2 \right)^{1/2}$$

where $\gamma_p > 0$ as it should be.

Inserting (36) in eq. (27) we obtain the ‘QED’ contribution to the asymmetry

$$\Delta(B+L)'_{SP}{}^{QED} \simeq 0.92 \frac{(2\pi)^3}{T^3} \Gamma \theta \sum_{ij}' c_{ij} I^{QED}(t_2) \quad (39)$$

where, now,

$$\begin{aligned}
I^{QED}(t_2) = & -\frac{2\pi}{3}T^2 \left(1 - \frac{v_w}{2} \log \frac{1+v_w}{1-v_w}\right) \frac{q_i q_j}{\sum_k (q_k)^2} \\
& \times \begin{cases} 0 & (t_2 < -\frac{\Delta z}{v_w}) \\ \left[t_2 + \frac{\Delta z}{v_w} + \cos \omega_p \left(t_2 + \frac{\Delta z}{v_w} \right) \frac{e^{-\gamma_p(t_2 + \Delta z/v_w)}}{\gamma_p} \right] & \left(-\frac{\Delta z}{v_w} < t_2 < 0 \right) \\ \left[\frac{\Delta z}{v_w} - \frac{e^{-\gamma_p t_2}}{\gamma_p} \left(\cos \omega_p t_2 - e^{-\gamma_p \Delta z/v_w} \cos \omega_p \left(t_2 + \frac{\Delta z}{v_w} \right) \right) \right] & (t_2 > 0), \end{cases}
\end{aligned} \tag{40}$$

The same considerations about the case $t_2 > 0$ made after eq. (32) apply also here. We can notice that photon exchange gives two different types of contributions. The first one has the same behaviour of that in (32), *i.e.* a linearly increasing asymmetry from $t_2 = -\Delta z/v_w$ to $t_2 = 0$. On the other hand, the second term exhibits a well known feature of plasma physics, namely, plasma damped oscillations induced by an external perturbation [23]. The damping rate here is given by γ_p . In the case $-\Delta z/v_w < t_2 < 0$, we see that the oscillating term dominates over the linear one only for $t_2 \rightarrow -\Delta z/v_w$, and is rapidly damped as $t_2 \rightarrow 0$, since $\exp(-\gamma_p \Delta z/v_w) \simeq \exp(-T \Delta z) \simeq \exp(-40)$. When $t_2 > 0$ the amplitude of the oscillation is always suppressed at least by a factor $10^{-1} \div 10^{-2}$ with respect to the linear one, and then we can conclude that the effect of the oscillating term is negligible unless t_2 is very near to $-\Delta z/v_w$.

An interesting feature of our results (32) and (40) can be appreciated if we calculate, by means of eq. (20), the electric charge Q' induced by the phase $\dot{\vartheta}$, taking into account both the direct contribution (29) and the ‘QED’ one (34) to \widetilde{D}_{ij}^R . It is easy to see that it is given by

$$\begin{aligned}
\langle Q' \rangle \propto & \left[\sum_i' y_i q_i \left(\sum_k q_k^2 - \sum_k' q_k^2 \right) \times \text{“linear contribution”} \right] \\
& + \text{“damped contribution”},
\end{aligned} \tag{41}$$

then, when *every* fermion is in equilibrium, the linear contribution to the thermal average of Q' vanishes, and we are left with the damped one. The reason is that in this case Q' coincides with the total fermion electric charge, and this is perfectly screened as in the usual QED plasma. Since in the real situation not all the fermions are in equilibrium, the linear contribution to (41) does not cancel. Anyway, this considerations are valid for electric charge only, while the other interesting charges, $(B+L)'$, $(B-L)'$, and, BP' , would have non zero linear contributions even if all the fermions were in equilibrium.

Putting all together, and neglecting the damped contribution, we find the final asymmetry in $(B + L)'$

$$\Delta(B + L)'_{SP} = -0.92 \frac{37}{240} (2\pi)^4 \frac{\Gamma\theta}{T} \left(1 - \frac{v_w}{2} \log \frac{1 + v_w}{1 - v_w}\right) \left(t_2 + \frac{\Delta z}{v_w}\right), \quad (42)$$

where we have assumed that the sphalerons turn off when the phase is still active ($-\Delta z/v_w < t_2 < 0$). Recalling that $\theta = \Delta\vartheta v_w/\Delta z$, and assuming that sphalerons cease to be active after a time interval $t_2 + \Delta z/v_w = f\Delta z/v_w$ from the turning on of $\dot{\vartheta}$ we get

$$\Delta(B + L)'_{SP} \simeq -2.3 \cdot 10^2 kT^3 \alpha_w^4 \Delta\vartheta f \quad (43)$$

where we have taken the reasonable value $v_w \simeq 0.2$ [19]. The above value is enhanced by nearly three orders of magnitude with respect to the original estimates by CKN [14] where transport phenomena were not taken into account.

The predicted baryon asymmetry of the Universe then comes out to be

$$\frac{\rho_B}{S} \simeq -10^{-6} k \Delta\vartheta f. \quad (44)$$

k is estimated in the range $0.1 \div 1$ from numerical simulations [22], while the value of f is still an open question. Following Dine and Thomas [16] we chose

$$f \simeq \frac{\alpha_w}{g} \simeq 5 \cdot 10^{-2}. \quad (45)$$

The observed baryon asymmetry, $\rho_B/S = (4 \div 7) \cdot 10^{-11}$, can then be reproduced for $\Delta\vartheta \simeq 10^{-2} \div 10^{-3}$, values which can be obtained either by explicit CP violation or by spontaneous CP violation at finite temperature [9] without entering in conflict with the experimental bounds on the electric dipole moment of the neutron.

The result (42) was obtained considering a charge potential of the form (10) where the sum extends on all the left handed fermions plus the right handed quark. Considering the more physical situation in which only the top quarks (left and right handed) feel the effect of $\dot{\vartheta}$ the coefficient $37/240$ in (42) should be changed into $9/32$, thus leading to an enhancement of a factor 1.8.

5. The effect of QCD sphalerons.

It is well known that the axial vector current of QCD has a triangle anomaly, therefore one can expect axial charge violation due to topological transitions analogous to the sphaleronic

transitions of the electroweak theory. The rate of these processes at high temperature may be estimated as [29, 15]

$$\Gamma_{strong} = \frac{8}{3} \left(\frac{\alpha_s}{\alpha_W} \right)^4 \Gamma_{SP} = \frac{8}{3} k (\alpha_s T)^4 \quad (46)$$

where α_s is the strong fine structure constant, leading to a characteristic time of order

$$\tau_{strong} = \frac{1}{192 k \alpha_s^4 T}. \quad (47)$$

Since $k \simeq 0.1 \div 1$ [22], we see that τ_{strong} is comparable to the time of passage of the bubble wall, and might also be smaller.

Recently, Giudice and Shaposhnikov have analyzed the effect of these ‘QCD sphalerons’ on the adiabatic baryogenesis scenario. They showed that, as long as these transitions are in equilibrium and fermion masses are neglected, no baryon asymmetry can be generated. Thus, the final result will be suppressed by a factor $\sim (m_{top}(T)/\pi T)^2$. In this paragraph we will reconsider the issue taking transport phenomena into account.

The effect of QCD sphalerons may be represented by the operator

$$\Pi_{i=i}^3 (u_L u_R^\dagger d_L d_R^\dagger)_i \quad (48)$$

where i is the generation index. Assuming that these processes are in equilibrium, we get the following chemical potentials equation

$$\sum_{i=1}^3 (\mu_{u_L^i} - \mu_{u_R^i} + \mu_{d_L^i} - \mu_{d_R^i}) = 0. \quad (49)$$

Eq. (49) contains the chemical potentials for *all* the quarks, and imposes that the total *right-handed* baryon number is equal to the total *left-handed* one. Using eqs. (1) and (2) we can rewrite it as

$$4\mu_{u_L} + \mu_{t_L} - \mu_{b_R} - 2\mu_{d_R} - 2\mu_{u_R} - 3\mu_{W^+} = 0, \quad (50)$$

where $\mu_{u_{L,R}} \equiv 1/2 \sum_{i=1}^2 \mu_{u_{L,R}^i}$, and $\mu_{d_R} \equiv 1/2 \sum_{i=1}^2 \mu_{d_R^i}$. One of the three new chemical potentials, μ_{b_R} , μ_{d_R} , and μ_{u_R} , can be eliminated using eq. (50), while the remaining two correspond to two more conserved charges that must be taken into account besides Q' , BP' , and $(B-L)'$ (now the primes mean that the summation has to be performed on right handed quarks too, but not on right handed leptons). We can choose

$$X \equiv \sum_{i=1}^3 d_R^i - \frac{3}{2} \sum_{i=1}^2 u_R^i, \quad (51)$$

$$Y \equiv b_L + t_L + t_R + \frac{1}{2} \sum_{i=1}^2 u_R^i - \sum_{j=1}^3 (e_L^j + \nu_L^j), \quad (52)$$

corresponding respectively to A_3 and A_2 in the notation of ref. [15]. Following the usual procedure, we can now express the abundance of any particle number at equilibrium as a linear combination of Q' , $(B - L)'$, BP' , X , and Y . For $(B + L)'$ we obtain the result

$$(B + L)'_{EQ} = -\frac{1}{5}(B - L)', \quad (53)$$

to be compared to eq. (5), which we obtained considering QCD sphalerons out of equilibrium. Thus, the equilibrium value for $(B + L)'$ depends only on the density of $(B - L)'$, in agreement with what was obtained in ref. [15]⁶. If transport phenomena were not present, as it was assumed in ref. [15], we could set $(B - L)'$ to zero and then conclude that QCD sphalerons allow no non vanishing $(B + L)'$ density, at equilibrium and in the massless approximation. On the other hand, including transport effects, we can easily calculate the $(B - L)'$ density induced by $\dot{\vartheta}$ using eq. (20), and then, through (53), the equilibrium value $(B + L)'_{EQ}$, which, unlike in ref. [15], comes out to be non vanishing inside the bubble wall. However this is not sufficient to conclude that we will have a non zero final baryon asymmetry when the bubble wall has passed by. As we discussed in Sect. 2., the generation of baryons inside the bubble walls is described by eq. (9), with the initial condition $(B + L)'_{SP} = 0$. Then we must calculate $(B + L)'_{TR}$, *i.e.* the contribution to $(B + L)'$ due to transport. If all the particles in equilibrium participated to the charge potential then we would find

$$(B + L)'_{TR}(t_x, \vec{x}) = (B + L)'_{EQ}(t_x, \vec{x}) \quad (54)$$

so that the system would always be on the minimum of the free energy inside the bubble wall and there would be no bias of the (electroweak) sphaleronic transitions. As a consequence, $(B + L)_{SP}$ would remain zero and no asymmetry would survive after the passage of the wall up to fermion mass effects, in agreement with what was found in ref. [15].

On the other hand, including only left and right handed top quarks into the charge potential, eq. (54) is no more satisfied and a non zero result for the final baryon asymmetry is recovered. In this case, the factor $37/240$ in eq. (42) should be changed to $25/72$.

6. Conclusions

In this paper we have analyzed the impact of transport phenomena on the so called ‘spontaneous baryogenesis’ mechanism of Cohen, Kaplan, and Nelson. We have assumed that inside

⁶Incidentally, note that this is not a general property due to the insertion of QCD sphalerons into the set of processes in equilibrium, but is due to the fact that only top Yukawa interactions are fast. If, for instance, bottom quark Yukawa interactions were also fast, then we would find that $(B + L)'_{EQ}$ is not simply proportional to $(B - L)'$, but depends also on the values of the other charges in equilibrium.

the walls of the bubbles nucleated during the electroweak phase transition a space time dependent ‘charge potential’ for (partial) fermionic hypercharge is generated. We stress again that no discussion about the origin of the charge potential has been given; in particular, since in the limit in which all the Yukawa couplings go to zero there is no communication between the Higgs and the fermion sectors, we expect that in this limit also the charge potential should go to zero. In the traditional approach of CKN there is no trace of this behaviour, and moreover we have shown that the results change in a sensible way according to the precise form of the charge potential which is considered. We reserve a discussion on this subject for a forthcoming publication. Our main interest here was to set a scheme for calculations in the the adiabatic scenario in the case in which such a charge potential is present, in order to determine the variations in the thermal averages induced by transport effects and the production of baryon number by sphalerons inside the bubble walls.

The main physical point of the paper is that the system should be regarded as a collection of subsystems in local equilibrium, in which the thermal averages for the conserved charges are not zero but are driven to non vanishing values by transport phenomena. In particular, the local equilibrium configurations will correspond to non zero values for $(B+L)'$. We have determined the local equilibrium configuration by means of a chemical potential analysis, calculating the values of the thermal averages for the conserved charges by using linear response theory. We have considered only the dominant contribution to these averages, in particular, we have neglected any fermion mass effect and also the coupling of the Higgs field to the Chern-Simons number.

We find that, in contrast with previous claims [17], the presence of transport phenomena does not prevent baryon number generation inside the bubble walls. The main source of disagreement with JPT is the following. In their paper, JPT consider the rate equation in the form

$$\dot{B} = -\frac{\Gamma_{SP}}{2T} (3\mu_{t_L} + 3\mu_{b_L} + \mu_{\tau_L} + \mu\nu_\tau), \quad (55)$$

where the term on the right hand side has been obtained by considering the variation of the free energy of the system due to a ‘sphaleron-like’ transition involving only the third generation, *i.e.* due to the processes $t_L t_L b_L \tau_L \leftrightarrow 0$ and $t_L b_L b_L \nu_\tau \leftrightarrow 0$. Then these authors impose that the chemical potential of any particle is proportional to the value of its hypercharge, and so they find that the right hand side vanishes as a consequence of the conservation of hypercharge (and of fermion hypercharge) in any sphaleronic transition. The point is that, assuming local equilibrium of the fast interactions, the chemical potentials of the single particle species *are not* proportional to their hypercharge. In fact, since the single particle numbers are not conserved quantities of the system, their abundances are reprocessed by fast interactions as to obtain their local equilibrium values. On the other hand, imposing that the particle chemical potentials are

proportional to hypercharge, would be equivalent to freeze out any interaction inside the bubble wall, both the fast and the slow ones, leaving transport phenomena as the only relevant process.

Also, if a charge potential for fermionic or total hypercharge is present, transport phenomena allow the generation of the baryon asymmetry even in the limit in which the Higgs VEV's go to zero. The reason is again that the thermal averages for Q' , $(B - L)'$ and BP' are non vanishing and then a $(B + L)$ asymmetry can be generated even if the electroweak symmetry is unbroken. This is strictly analogous to the well known result of the survival of a $B + L$ asymmetry when a $B - L$ density, eventually of GUT origin, is present [20]. Of course, in the limit in which the VEV's go to zero, also the charge potential should go to zero, since no complex phase can emerge from the Higgs sector in this case. Then, also this suppression, like the one due to vanishing h_{top} , should be made evident by an accurate discussion on the origin of the charge potential. As a consequence, our results for the baryon asymmetry, eq. (43) should be probably multiplied by a further suppression factor roughly of order $(m_{top}(T)/T)^2$.

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